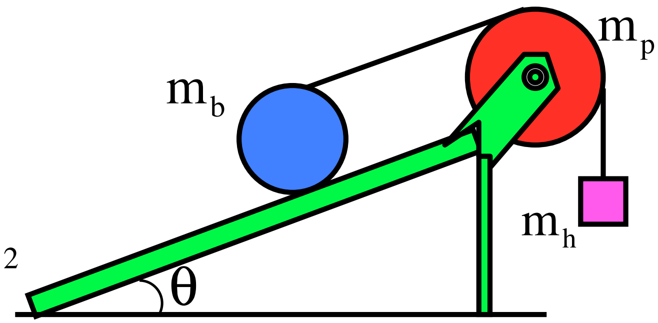
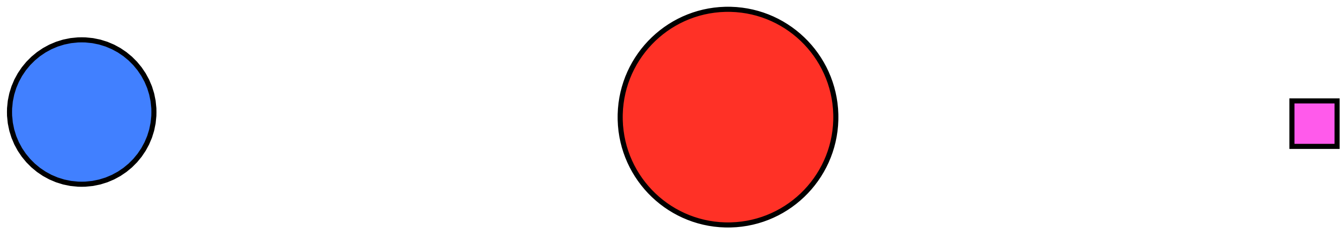
10.) A hanging mass is attached to a string which is threaded over a massive pulley of radius R and wound around a ball of radius R/2 sitting on an incline. The ball and pulley have the same radius. You additionally know:



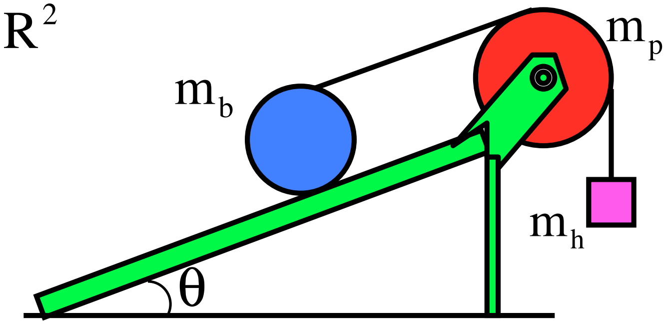
$$m_b,\ m_h,\ m_p,\ R,\ g,\ \theta,\ I_{cm\ of\ pulley} = \frac{1}{2}m_pR^2,\ \text{and}\ I_{cm\ of\ ball} = \frac{1}{6}m_bR^2$$

a.) Ignoring the forces acting at the pulley's pin, draw a f.b.d. identifying all the forces acting on both masses and the pulley.



b.) Derive an expression for the hanging mass's *upward* acceleration.

m_b, m_h, m_p, R, g, θ, $I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2$, and $I_{\text{cm of ball}} = \frac{1}{6} m_b R^2$



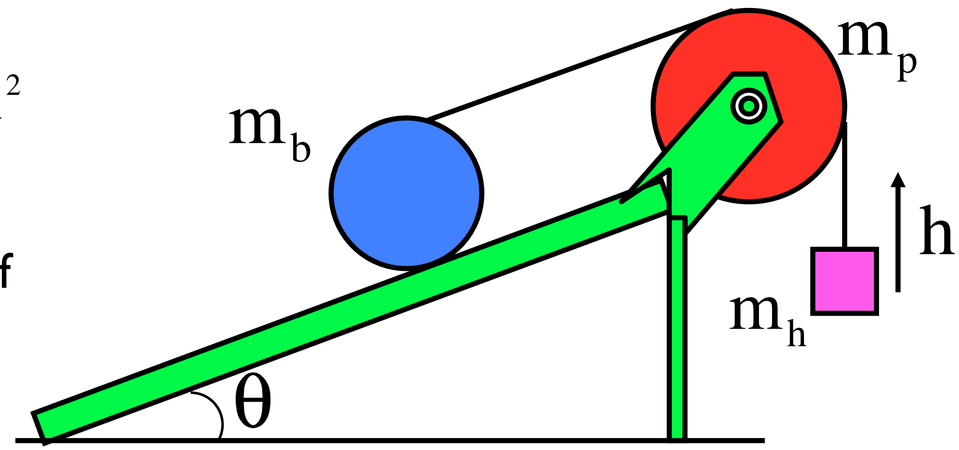
b.) (con't.)

c.) What is the pulley's angular acceleration?

$m_b, m_h, m_p, R, g, \theta,$

$$I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2, \text{ and } I_{\text{cm of ball}} = \frac{1}{6}m_bR^2$$

d.) The hanging mass rises from rest a distance "h." What is its *velocity magnitude* at the top of the rise?



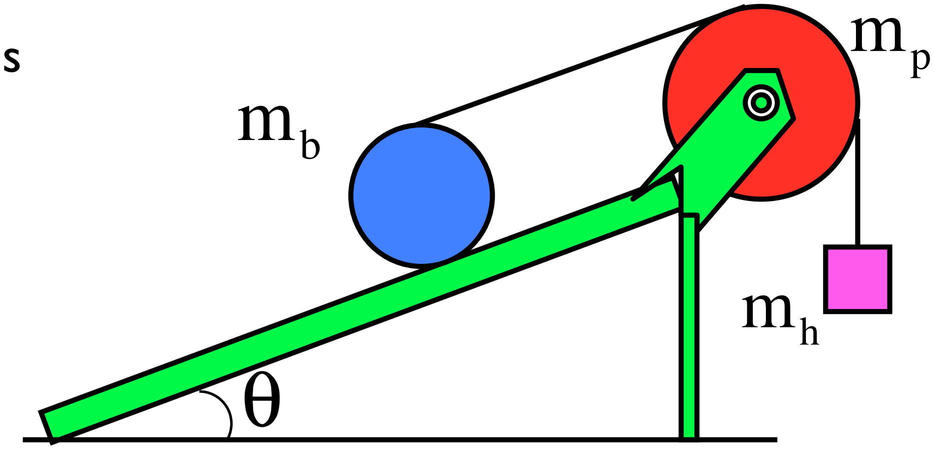
e.) What is the *angular velocity* of the pulley at that point?

f.) What is the *angular momentum* of the pulley at that point?

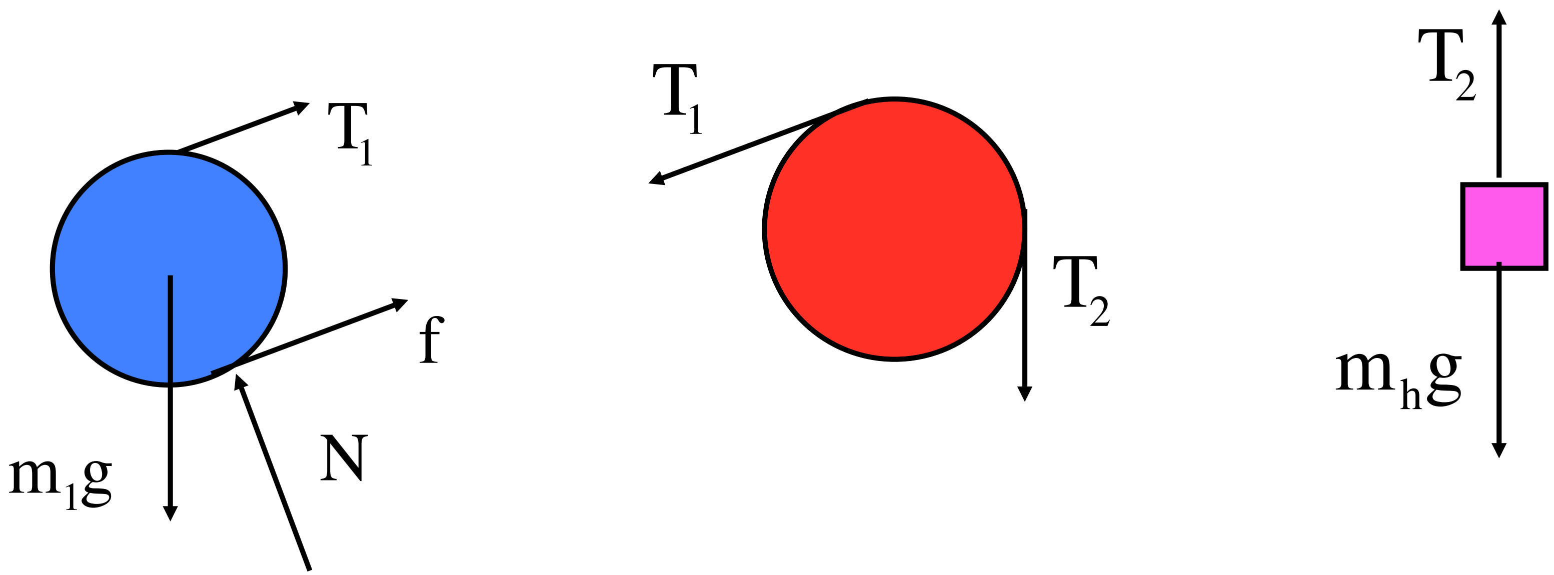
10.) A hanging mass is attached to a string which is threaded over a massive pulley of radius R and wound around a ball of radius R/2 sitting on an incline. The ball and pulley have the same radius. You additionally know:

$$m_b, m_h, m_p, R, g, \theta,$$

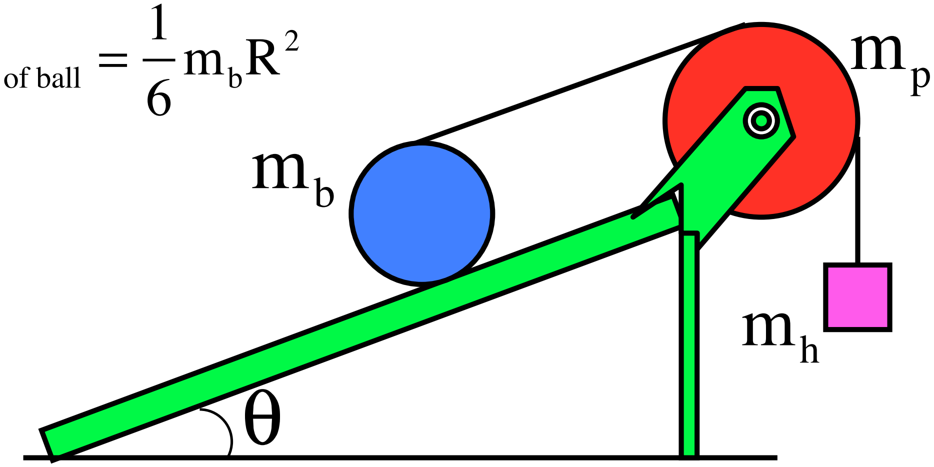
$$I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2, \text{ and } I_{\text{cm of ball}} = \frac{1}{6} m_b R^2$$



a.) Ignoring the forces acting at the pulley's pin, draw a f.b.d. identifying all the forces acting on both masses and the pulley.



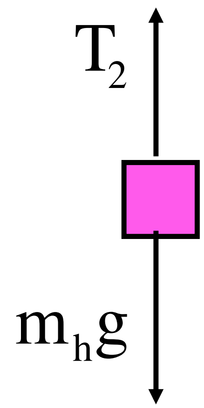
$$m_b, m_h, m_p, R, g, \theta, I_{cm\ of\ pulley} = \frac{1}{2}m_pR^2, \text{ and } I_{cm\ of\ ball} = \frac{1}{6}m_bR^2$$



b.) Derive an expression for the hanging mass's *upward* acceleration.

This is clearly a Newton's Law problem, so I'd probably start with the easiest piece to deal with. That would be the hanging mass. Doing a free body diagram and summing the forces yields:

m_h



$$\sum F_{m_h,y}$$

$$T_2 - m_hg = m_ha_h$$

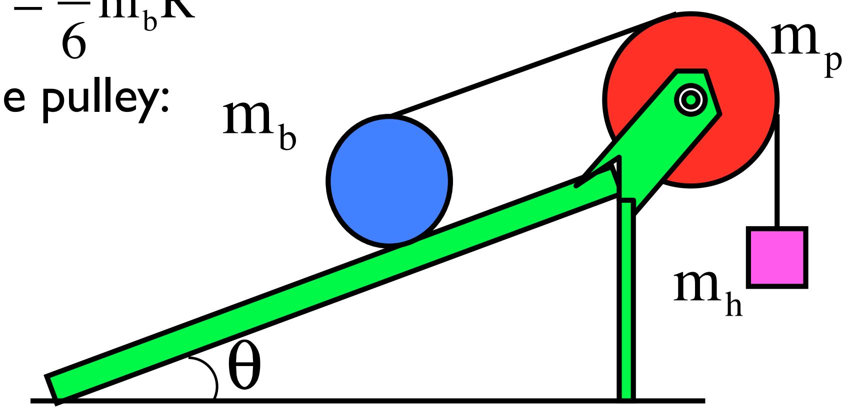
$$\Rightarrow \quad T_2 = m_hg + m_ha_h$$

There is a subtlety here that doesn't make a lot of difference with this part of the calculation but is going to be important later. It is the observation that the hanging mass's acceleration is also the acceleration *of the string*. As such, I'm going to rewriting my conclusion as:

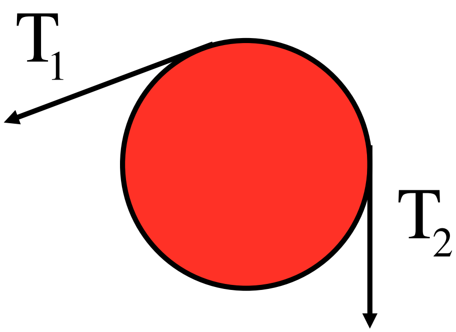
$$T_2 = m_hg + m_ha_{string}$$

$m_b,\ m_h,\ m_p,\ R,\ g,\ \theta,\ I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$, and $I_{\text{cm of ball}} = \frac{1}{6}m_bR^2$

We need another expression with T_2 in it. Let's try the pulley:



pulley



$$\sum \Gamma_{\text{pulley}}$$

$$-T_2R + T_1R = +I_{pin}\alpha_{pully}$$

$$\Rightarrow \quad -T_2\cancel{R} + T_1\cancel{R} = \left(\frac{1}{2}m_p\cancel{R^2}\right)\left(\frac{a_{\text{string}}}{\cancel{R}}\right)$$

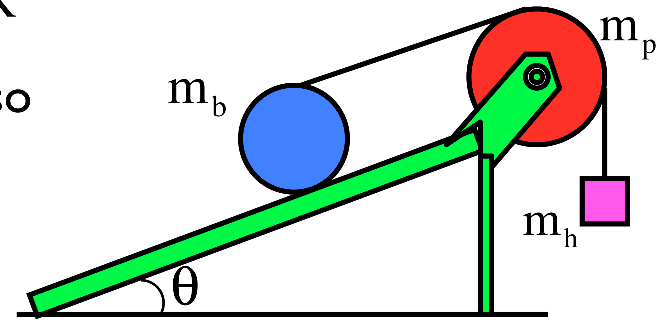
$$\Rightarrow \quad -T_2 + T_1 = \frac{1}{2}m_pa_{\text{string}}$$

$$\Rightarrow \quad -\left(m_hg + m_ha_{\text{string}}\right) + T_1 = \frac{1}{2}m_pa_{\text{string}}$$

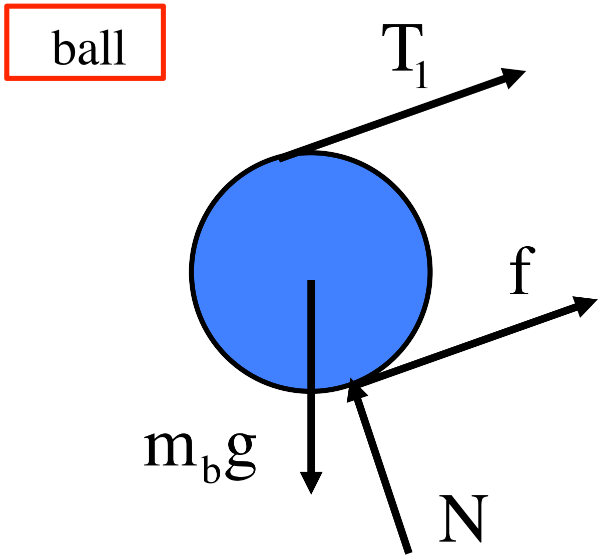
$$\Rightarrow \quad T_1 = \left(\frac{1}{2}m_p + m_h\right)a_{\text{string}} + m_hg$$

We now need an expression for T_1

$m_b,\ m_h,\ m_p,\ R,\ g,\ \theta,\ I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$, and $I_{\text{cm of ball}} = \frac{1}{6}m_bR^2$



We can get an expression for T_1 by looking at the ball. Doing so yields:



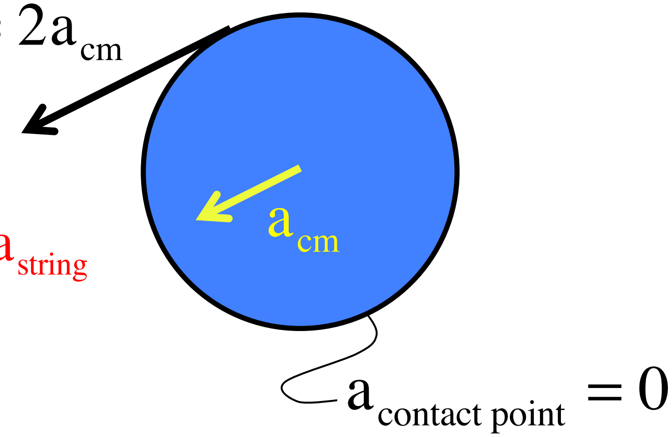
$\sum F_x$

$$f + T_1 - m_b g \sin\theta = -m_b a_{cm}$$

We now need an expression for the frictional force *f*, AND we need to relate the acceleration of the *center of mass of the ball* and the *acceleration of the string*.

The string part is easy (see sketch to right).

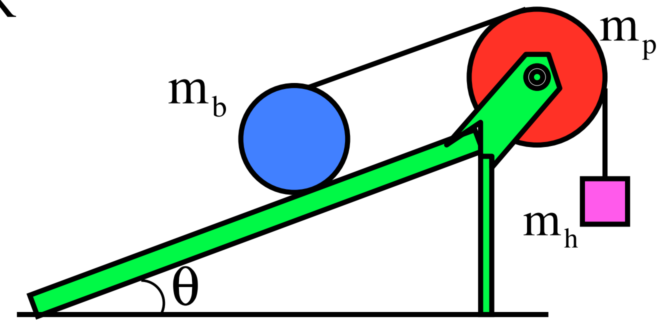
$a_{string} = 2a_{cm}$



$$\Rightarrow\ a_{cm} = \frac{1}{2}a_{string}$$

$$m_b, m_h, m_p, R, g, \theta, I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2, \text{ and } I_{\text{cm of ball}} = \frac{1}{6}m_bR^2$$

With that, we can re-write the force expression as:



$$\sum F_x$$

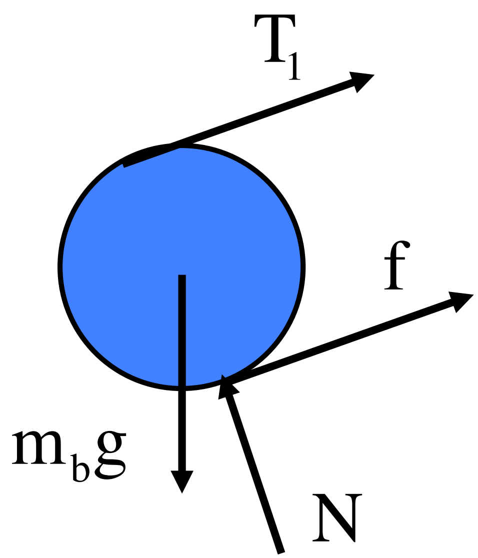
$$f + T_1 - m_b g\sin\theta = -m_b\left(\frac{a_{string}}{2}\right)$$

As for the frictional force *f* (and remembering that the ball's radius is R/2), we can sum the torque on the ball about the ball's *center of mass:*

$$\sum \Gamma_{ball,cm}$$

$$f\left(R/2\right) - T_1\left(R/2\right) = I_{ball,cm}\alpha_{ball}$$

$$f\left(R/2\right) - T_1\left(R/2\right) = \left(\frac{1}{6}m_bR^2\right)\alpha_{ball}$$



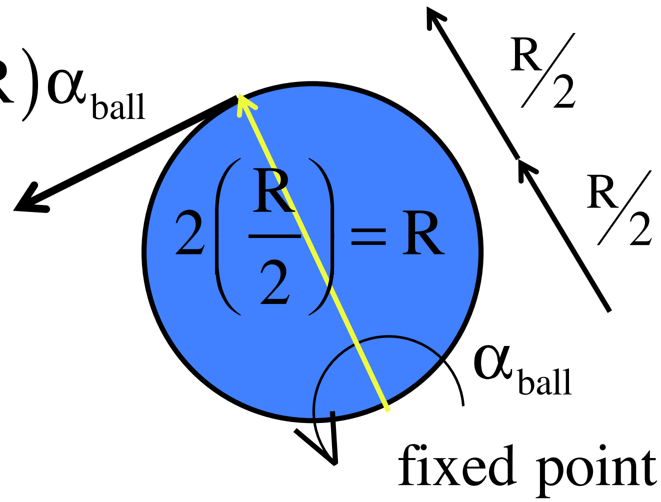
Another unknown: α_{ball}.

$m_b, m_h, m_p, R, g, \theta, I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$, and $I_{\text{cm of ball}} = \frac{1}{6}m_bR^2$

We now need an expression for α_{ball}.

$$a_{string} = (R)\alpha_{ball}$$

$$\Rightarrow \alpha_{ball} = \frac{a_{string}}{R}$$



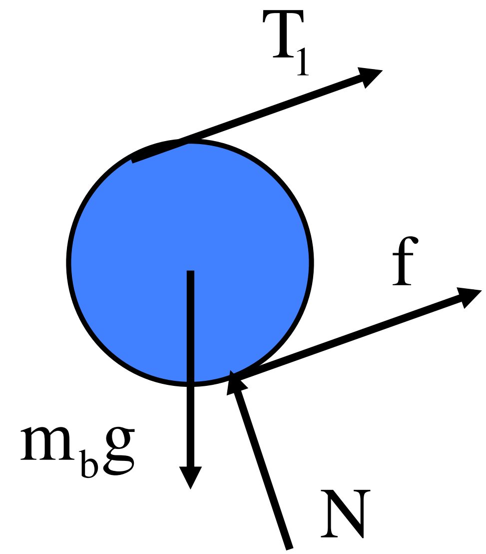
This allows us to write the torque calculation as:

$$\sum \Gamma_{ball,cm}$$

$$f\left(R/2\right) - T_1\left(R/2\right) = +I_{ball,cm}\alpha_{ball}$$

$$f\left(R/2\right) - T_1\left(R/2\right) = \left(\frac{1}{6}m_bR^2\right)\left(\frac{a_{string}}{R}\right)$$

$$\Rightarrow \quad f = T_1 + \left(\frac{1}{3}m_b\right)a_{string}$$



$$m_b,\ m_h,\ m_p, R, g, \theta, I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2,\ \text{and } I_{\text{cm of ball}} = \frac{1}{6}m_bR^2$$

Putting 1 into 2 and 1 into 3, then equating 2 and 3 yields:

$$T_1 = \left(\frac{1}{2}m_p + m_h\right)a_{string} + m_hg$$

$$f = T_1 + \left(\frac{1}{3}m_b\right)a_{string} \quad \Rightarrow \quad f = \left[\left(\frac{1}{2}m_p + m_h\right)a_{string} + m_hg\right] + \left(\frac{1}{3}m_b\right)a_{string}$$

$$= \left(\frac{1}{2}m_p + m_h + \frac{1}{3}m_b\right)a_{string} + m_hg$$

$$f + T_1 - m_bg\sin\theta = -m_b\left(\frac{a_{str}}{2}\right) \quad \Rightarrow$$

$$\left[\left(\frac{1}{2}m_p + m_h + \frac{1}{3}m_b\right)a_{string} + m_hg\right] + \left[\left(\frac{1}{2}m_p + m_h + \frac{1}{3}m_b\right)a_{string} + m_hg\right] - m_bg\sin\theta = -m_b\left(\frac{a_{str}}{2}\right)$$

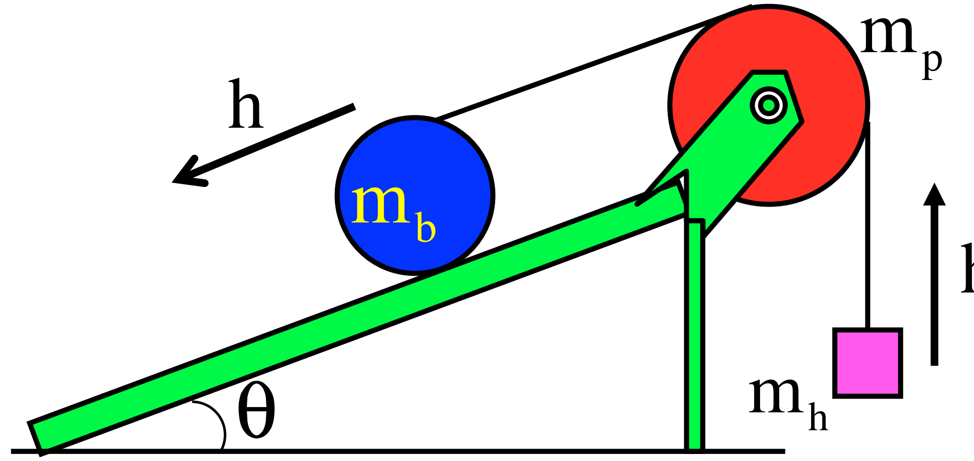
$$\Rightarrow \quad \left(m_p + 2m_h + \frac{2}{3}m_b\right)a_{string} + m_hg(1-\sin\theta) = -m_b\left(\frac{a_{strring}}{2}\right)$$

$$\Rightarrow \quad +m_hg(1-\sin\theta) = -\left(\frac{m_b}{2} + m_p + 2m_h + \frac{2}{3}m_b\right)a_{string}$$

$$\Rightarrow \quad a_{string} = -\frac{m_hg(1-\sin\theta)}{\left(\frac{m_b}{6} + m_p + 2m_h\right)}$$

c.) What is the pulley's angular acceleration?

$$a_{string} = R\alpha_{pulley}$$



d.) The hanging mass rises from rest a distance "h." What is its velocity magnitude at the top of the rise?

If the hanging mass accelerates from rest *upward*, the ball on the incline must go

DOWN the incline. That means it initially has potential energy while the hanging mass doesn't.

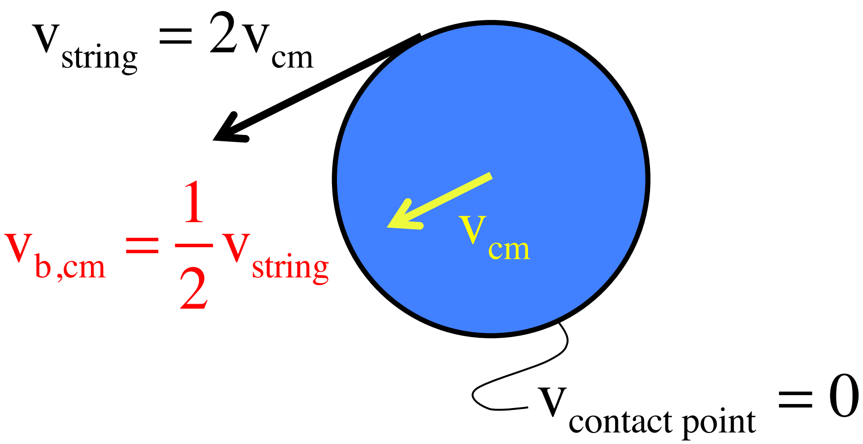
$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + \left[m_b g(h\sin\theta)\right] + 0 = \left[\frac{1}{2}m_b v_{b,cm}{}^2 + \frac{1}{2}I_{b,cm}\omega_b{}^2 + \frac{1}{2}m_h v^2 + \frac{1}{2}I_{pulley}\omega_p{}^2\right] + \left[m_h g h\right]$$

All we need now is a relationship between the ball's *center of mass* velocity and the velocity of the string and hanging mass.

d.) (con't.)

Relationship between ball's c. of m. velocity and string velocity:



$$\Rightarrow v_{b,cm} = \frac{1}{2} v_{string}$$

Leaving us with:

$$\left[m_b g(h\sin\theta)\right] = \left[\frac{1}{2} m_b v_{b,cm}^2 + \frac{1}{2} m_h v_s^2 + \frac{1}{2} I_{pulley} \omega^2\right] + \left[m_h g h\right]$$

$$\left[m_b g(h\sin\theta)\right] = \left[\frac{1}{2} m_b \left(\frac{v_s}{2}\right)^2 + \frac{1}{2} m_h v_s^2 + \frac{1}{2}\left(\frac{1}{2} m_p R^2\right)\left(\frac{v_s}{R}\right)^2\right] + \left[m_h g h\right]$$

$$\Rightarrow \quad v = \sqrt{\frac{2m_b g(h\sin\theta) - 2m_h g h}{\frac{m_b}{4} + m_h + \frac{m_p}{2}}}$$

$$m_b,\ m_h,\ m_p, R, g, \theta, I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2, \text{ and } I_{\text{cm of ball}} = \frac{1}{6} m_b R^2$$

e.) What is the *angular velocity* of the *pulley* at that point?

$$\omega = \frac{v_s}{R}$$

f.) What is the *angular momentum* of the *pulley* at that point?

$$L = I_{pin}\omega$$